of the Council, at a short interval of time, deprecating further delay, and censuring what had already occurred.

One name in the list of contributors from the town of Cambridge, that of Hobson the carrier, celebrated by some lines of Milton, has, from that circumstance, attracted attention, from the sum demanded and the nature of his occupation.

" The original papers have been injured by damp, and rendered in some degree, but not materially, defective.

" In order to render the papers more accessible for perusal, I have sketched out a table of reference which I enclose, and avail myself of the kindness of my valued friend, the Lord Bishop of Cashel, for their transmission to Dublin.

<div style="text-align: center">" I am, Sir,</div>

<div style="text-align: center">" Your obedient Servant,</div>

<div style="text-align: center">" JOHN NEWPORT.</div>

" *Sir William R. Hamilton,*

  President R. I. Academy,

    &c. &c. &c."

The special thanks of the Academy were voted to Sir J. Newport.

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A paper was read by William Roberts, Esq., F. T. C. D., " on the Rectification of Lemniscates and other Curves."

Let a curve be traced out by the feet of perpendiculars dropped from a fixed origin upon the tangents to a given curve: and from this new curve, let another be derived by a similar construction, and so on. Also let a curve be imagined which is constantly touched by perpendiculars to the radii vectores of the given curve, drawn at the points where it is met by these radii, and from this let another be derived by a similar mode of generation, and so on.

Then if s_n denote the arc of the curve which is n^{th} in order in the former series, and s_{-n} that of the nth in the latter, we shall have

$$ds_{\pm n} = \frac{\pm\, nr\dfrac{d^2\omega}{dr^2} + (1\pm n)\dfrac{d\omega}{dr} + r^2\dfrac{d\omega^3}{dr^3}}{\left(1 + r^2\dfrac{d\omega^2}{dr^2}\right)^{\frac{\pm n + 1}{2}}} \left(r\frac{d\omega}{dr}\right)^{\pm n - 1} r\,dr,$$

$\textsc{f}\,(r, \omega) = 0$ being the polar equation of the given curve.

It is convenient to distinguish the curves of the two series by calling those of the former *positive*, and those of the latter *negative*; we may also generally denote their polar coordinates by the symbols $r_{\pm n}$, $\omega_{\pm n}$.

If the given curve, which may be denominated the base of either system, be an ellipse whose centre is the origin, it will be found, by applying the above formula, that the negative curves will in general have their arcs expressible by elliptic integrals of the first and second kinds, whose modulus is the eccentricity of the base-ellipse. The arc of the first will involve only a function of the first kind: a result which has been given by Mr. Talbot in a letter addressed to M. Gergonne, and inserted in the *Annales des Mathematiques*, tom. xiv. p. 380.

A function of the third kind, with a circular parameter $- 1 + b^4$, where b is the semiaxis minor of the ellipse, its semiaxis major being unity, and the modulus of which is the eccentricity, enters into the arcs of all the positive curves; and their general rectification depends only on that of the ellipse, and of the first derived, both positive and negative.

The quadrants of the ellipse, and of the first two curves, positive and negative, are connected by the following relation :

$$(s_{-1} + s_1)\, s_{-1} = (3\,s - s_{-2})\,(2\,s - s_2).$$

It is worthy of notice, that if the eccentricity be $\dfrac{\sqrt{5}-1}{2}$, the functions of the third kind disappear, and the rectification of both series depends only on that of the ellipse and of the first negative curve.

If the base curve be a hyperbola, whose centre is the origin, the arcs of all the curves of the negative series will depend only on elliptic functions of the first and second kinds. But the general expression for the arc in the positive series contains a function of the third kind, the parameter of which is alternately circular and logarithmic: the curves of an odd order involving the same function of the circular kind, and those of an even order the same of the logarithmic kind, if the real axis of the base-hyperbola be greater than the imaginary, and *vice versa*.

Mr. Roberts also shows, that besides the case of the equilateral hyperbola, in which the first positive curve is the lemniscate of Bernouilli, and which has been the only one hitherto noticed, at least as far as he is aware, there are two others, in which the arc of the first positive curve can be expressed by a function of the first kind, with the addition of a circular arc in one case, and of a logarithm in the other. The first of these occurs when the imaginary semiaxis is equal to $\dfrac{\sqrt{5}-1}{2}$ (the distance between the centre and focus being unity), and this fraction is the modulus of the function. The other case is furnished by the conjugate hyperbola, and the modulus is complementary. In both these cases functions of the third kind disappear from the arcs of the positive curves.

If the hyperbola be equilateral, and its semiaxis be supposed equal to unity, the general equation of the derived curves of both series may be presented under the form

$$r_{\pm n}^{\frac{2}{\pm 2n-1}} = \cos\left(\frac{2\omega_{\pm n}}{\pm 2n-1}\right).$$

The successive curves represented by this equation are very curiously related to each other. The following property appears worthy of remark:

Let P_{n-1}, P_n, P_{n+1} be corresponding points on the

$(n-1)^{th}$, n^{th}, and $(n+1)^{th}$ curves of the positive series respectively, and v their common vertex, which is also that of the hyperbola, then will

$$\text{arc } \text{vP}_{n-1} + \text{right line } \text{P}_{n-1}\,\text{P}_n = \frac{2n-1}{2n+1} \text{ arc } \text{vP}_{n+1}.$$

Mr. Roberts states that he has demonstrated the property in a manner purely geometrical.

This equation shows that the arcs of all the curves of an odd order will depend only on that of Bernouilli's lemniscate, or the function $\text{F}\left\{\sqrt{\tfrac{1}{2}}, \phi\right\}$, and those of an even order only on the arc of the second of the series. This latter arc is three times the difference between the corresponding hyperbolic arc and the portion of the tangent applied at its extremity, which is intercepted between the point of contact and the perpendicular dropped upon it from the centre : and the entire quadrant is three times the difference between the infinite hyperbolic arc and its asymptot.

Also, s_n, s_{n+1}, denoting the quadrants of the nth, and $(n+1)^{th}$ curves, the following very remarkable relation exists between them,

$$s_n\, s_{n+1} = (2n+1)\frac{\pi}{4}.$$

The curves of the negative series enjoy analogous properties.

Lastly, let the base curve be a circle, the origin being within it: and it appears that the rectification of the curves of both series, which are of an even order, can be effected by the arcs of circles; and that those of an odd order, which belong to the positive series, will involve elliptic integrals of the first and second kinds in their arcs. The negative curves of an odd order contain a term depending on a function of the third kind, which is however reducible to a function of the first kind and a logarithm.

By the particular consideration of the first negative curve in this case, Mr. Roberts was led to a very simple demon-

stration of the equation which results from the application of Lagrange's celebrated scale of reduction to elliptic functions of the second kind, and which is nothing more than the analytical expression of Landen's theorem.

Professor Mac Cullagh exhibited to the Academy some Roman Denarii, from the collection of Mrs. Alexander of Blackheath (Coleraine).

These coins (twenty-eight in number) were found in the year 1831, along with an immense quantity of others of the same kind, weighing altogether about eight pounds, by a labourer who was digging in a field on the Faugh Mountain, near Pleaskin, one of the headlands of the Giant's Causeway. According to an account published at the time in the Belfast News' Letter (June, 1831), and communicated to the Academy by the Rev. Dr. Drummond, they were found under a flat stone which was turned up by the spade. Nearly 200 of them (says this account) were sold for a trifling sum to an English gentleman at Coleraine, and some of the remainder were bought by the Rev. R. Alexander. Of the twenty-eight coins that were exhibited, only seventeen have their legends legible, and these are of the times of the emperors, from Vespasian to the Antonines. The following list of them has been supplied by Dr. Aquilla Smith, with references to the catalogue of the University Cabinet, published by the Rev. J. Malet, F.T.C.D.

1. Vespasian, . . . *Malet*, 384.
2. Vespasian, . . . Reverse, a winged Caduceus.
3. Domitian, . . . *Malet*, 452.
4. Domitian, . . . Reverse, Minerva.
5. Nerva, ... *Malet*, 467.
6. Trajan, ⎫
7. Trajan, ⎬ . . . *Malet*, 513.
8. Trajan, ... Reverse, Minerva.
9. Trajan, ... Reverse, a Female seated.
10. Hadrian, ... *Malet*, 548.